BCA-401 & CSA 5002T

Lecture Notes: Discrete Mathematics & Computing

Programme – BCA | Semester IV

- ☑ Unit I : Set Theory, Relations, Functions
- ☑ Unit II : Trigonometry
- ☑ Unit III: Cartesian Coordinate System
- ☑ Unit IV: Straight Lines ☑ Unit V: Circles
- ☑ Unit VI & VII: Polynomials & Quadratic Equations
- ☑ Unit IX & X: Determinants, Permutation & Combination

1. Set Theory: Introduction & Definitions (3.1 - 3.2.6)

Definition: **Set** is an **unordered collection** of objects. Object = element or member.

Representation Methods

1. **Roster / List Method:** Listed in braces.

$A = \{2, 3, 5, 7, 11, 13\}$

$N = \{2, 4, 6, \ldots\}$

2. **Set-Builder Method:** Described by property.

$A = \{ x : x \text{ is a prime number} < 15\}$

$B = \{ x : x = 2n, n \in N \}$

Types of Sets

- **Empty Set ($\varnothing$ or $\{\}$):** No elements. (Ex: Even primes > 10)
- **Singleton Set:** One element.
- **Finite Set:** Limited elements.
- **Infinite Set:** Unlimited elements.
- **Universal Set (U):** Contains all objects under consideration.
- **Equal Sets:** Same elements. (Ex: $A=\{2,5,7,9\}$ equals $B=\{5,2,7,9\}$)
- **Subset ($\subseteq$):** $A \subseteq B$ if every element of A is in B.
- **Superset ($\supseteq$):** $A \supseteq B$ if B is a subset of A.
- **Proper Subset ($\subset$):** Subset but not equal.

Operations on Sets (3.2.7 - 3.2.13)

Basic Operations

- **Union ($A \cup B$)**: Elements in A or B or both
  - Ex: $A=\{1,2,3,5,7\}$, $B=\{2,5,10,11\} \rightarrow A \cup B = \{1,2,3,5,7,10,11\}$
- **Intersection ($A \cap B$)**: Common elements
  - Ex: $A \cap B = \{2,5\}$
- **Difference ($A - B$)**: In A but not B
  - Ex: $A=\{1,4,7,8,9\}$, $B=\{4,9,11,13\} \rightarrow A-B=\{1,7,8\}$
- **Complement (A^c or A')**: Elements in U not in A

Cartesian Product

$A \times B = \{(a, b) : a \in A, b \in B\}$

Example: $A=\{1,2,3\}$, $B=\{4,5\}$

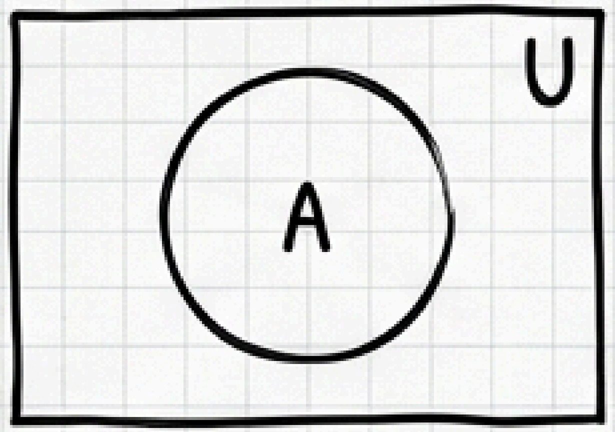
$A \times B = \{(1,4),(1,5),(2,4),(2,5),(3,4),(3,5)\}$

Quantitative Concepts

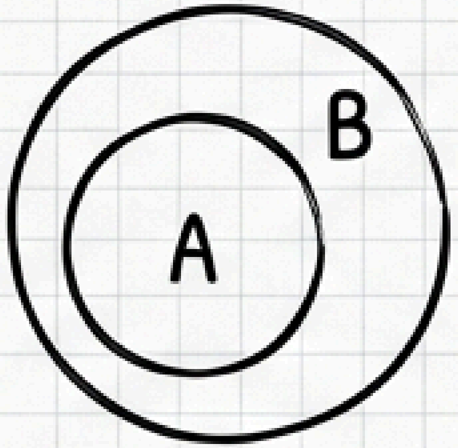
- **Cardinality ($|A|$)**: Total number of elements
  Ex: $|A|=6$
- **Power Set ($P(A)$)**: Set of all subsets
  Ex: $A=\{1,2,3\} \rightarrow P(A) = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$
- **Important**: If $|A| = n$, then $|P(A)| = 2^n$.

Diagrammatic Representation & Algebra of Sets

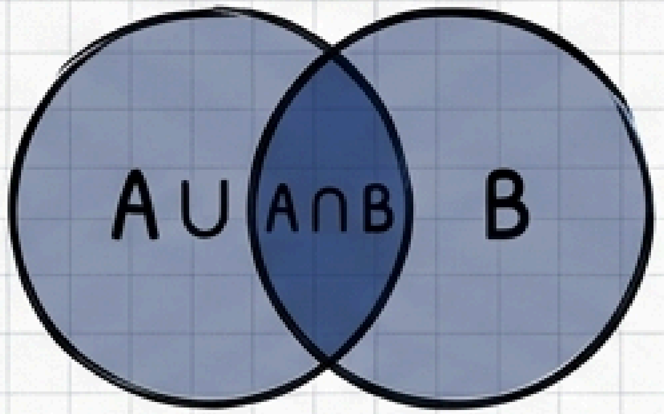
$A \subseteq U$



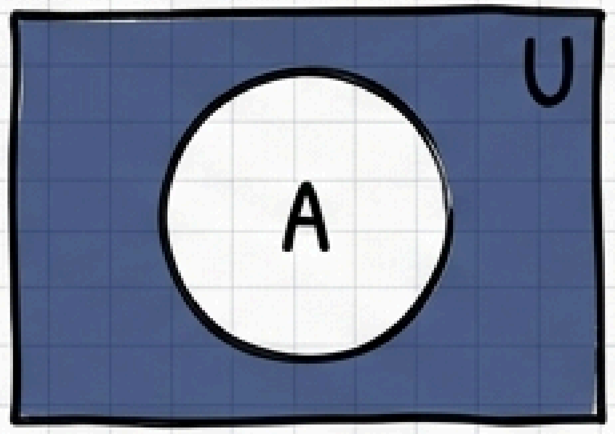
$A \subseteq B$



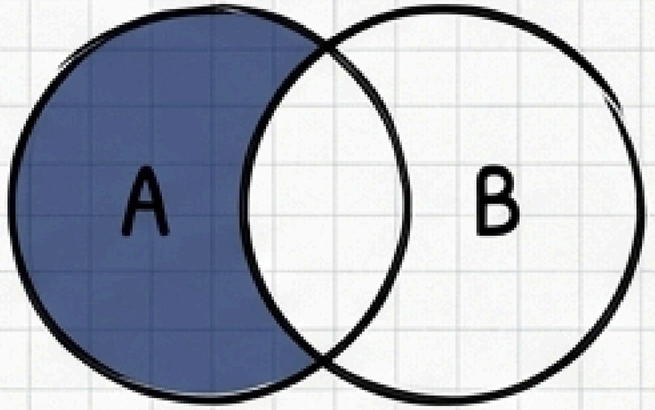
Union & Intersection



Complement A^c



Difference $A - B$



$A \subseteq U$ $A \subseteq B$ Union & Intersection Complement A^c Difference $A - B$

The Algebra of Sets (Laws)

1. Commutative: $A \cup B = B \cup A$; $A \cap B = B \cap A$
2. Associative: $(A \cup B) \cup C = A \cup (B \cup C)$
3. Identity: $A \cup \varnothing = A$; $A \cap U = A$
4. Universal/Null: $A \cup U = U$; $A \cap \varnothing = \varnothing$
5. Distributive: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
6. Complement: $A \cup A^c = U$; $A \cap A^c = \varnothing$
7. De Morgan's: $(A \cup B)^c = A^c \cap B^c$; $(A \cap B)^c = A^c \cup B^c$
8. Idempotent: $A \cup A = A$
9. Double Complement: $(A^c)^c = A$
10. Difference: $A - B = A \cap B^c$

3.5 Computer Representation & 3.6 Relations

Computer Representation of Sets

- Concept: Order elements of finite U ($a_1 \ldots a_n$). Represent subset A as bit string (1 if $a_i \in A$, 0 otherwise).

U={1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.
A={3, 5, 6, 9, 10}.

Bit String: 0 0 1 0 1 1 0 0 1 1

Introduction to Relations

- Definition 3.6.1: Binary relation R from A to B is a subset of A × B. Notation: aRb.
- Domain: First elements.
  Range: Second elements.
- Inverse Relation (R^{-1}): $\{(b,a) : (a,b) \in R\}$.
- Examples:
  - Ex 3.2: "is adjacent to" (Countries). (India, Nepal) ∈ R.
  - Ex 3.3: "m divides n". (6, 30) ∈ R.
  - Ex 3.5: If R has (1,x), R^{-1} has (x,1).

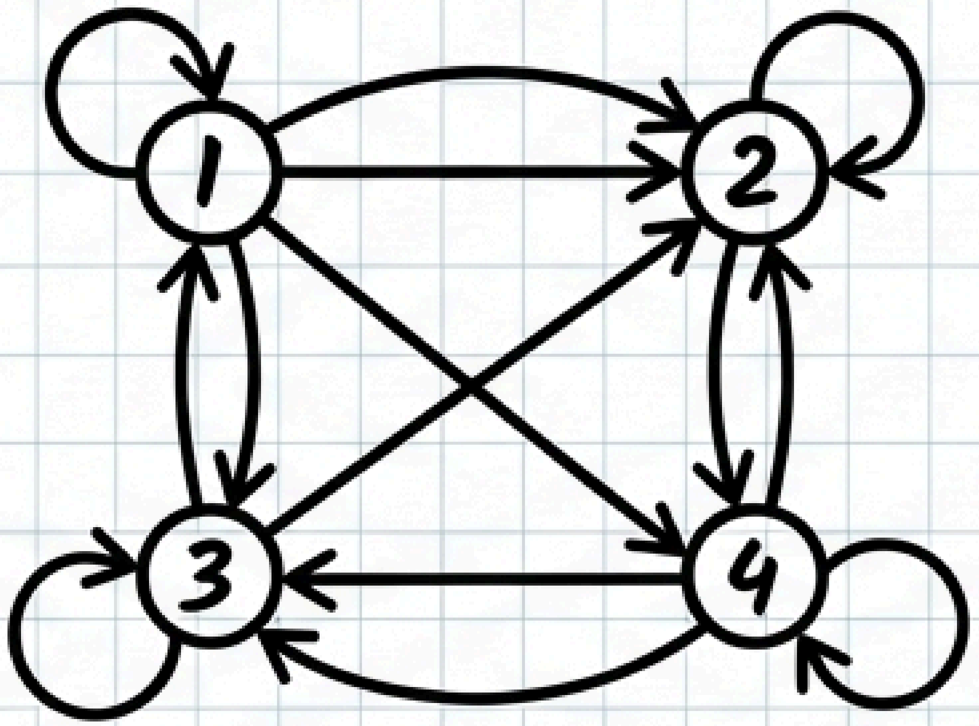
3.7 Representation & Composition of Relations

Matrix Representation (M_R)

	x	y	z
1	0	1	0
2	1	0	1
3	1	1	0
4	0	0	1

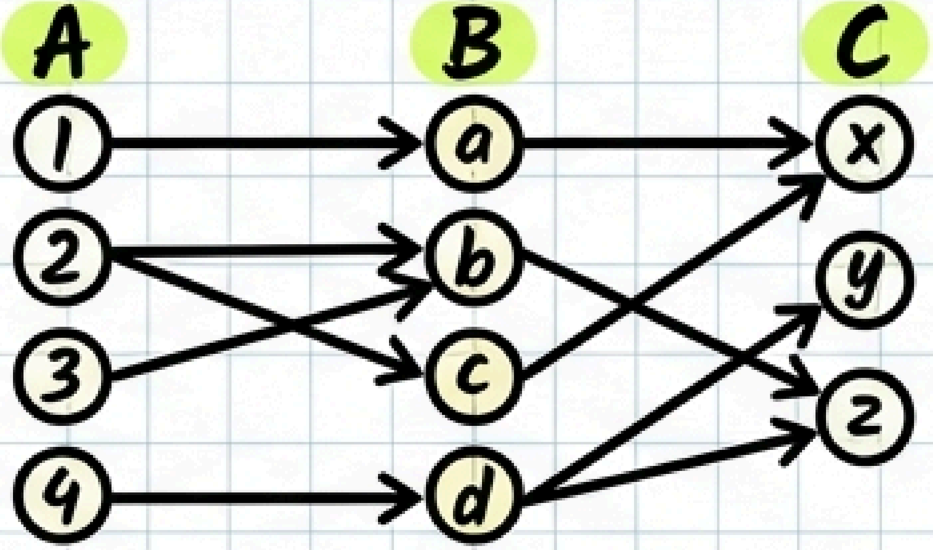
Note: $m_{ij} = 1$ if $a_i \, R \, b_j$

Digraph Representation



Composite Relation ($R \circ S$)

Definition: $a(R \circ S)c$ if $\exists b$ such that aRb and bSc.



Result: $R \circ S = \{(2,z),(3,x),(3,z)\}$

3.8 Types of Relations (Part I)

	Definition:	Visual Cues:	Example:
Reflexive	aRa for every $a \in A$.	Matrix diagonal = 1. Digraph = all nodes have loops.	'divides' is reflexive.
Irreflexive	a (not R) a for every $a \in A$.	Matrix diagonal = 0. No loops.	'less than' ($<$).
Symmetric	$aRb \Rightarrow bRa$.		'is equal to', $\{(1,2), (2,1)\}$.
Anti-symmetric	aRb and $bRa \Rightarrow a = b$.	**Note:** Never aRb and bRa if $a \neq b$.	'less than or equal to' ($\leq$).

Types of Relations (Part 2) & Partitions

Additional Types

- Asymmetric: If aRb, then b (not R) a. Ex: < on real numbers.
- Transitive: aRb, bRc ⇒ aRc. Ex: 'divides'.

EQUIVALENCE RELATION

Reflexive + Symmetric + Transitive. Ex: Parallel lines, Modulo m.

Partitions (3.9)

Definition: Collection P of disjoint non-empty sets (blocks) whose union is A.

- Valid Partition Ex 3.23: A={1,2,3,4,5}. A1={1,2}, A2={3,5}, A3={4}. (No overlap, union is A).
- Invalid Partition Ex 3.24: A2={3,5}, A3={4,5,6}. (Overlap at 5).

Theorem: Equivalence classes R(a) form a partition.

Examples & Unit End Exercises

Worked Example 3.24

Relation: $a - b \leq 2$ on $\mathbb{Z}^+$.

- Reflexive? YES ($0 \leq 2$)
- Irreflexive? NO (1-1=0)
- Symmetric? YES ($a-b \leq 2 \Rightarrow b-a \leq 2$)
- Asymmetric? NO
- Anti-symmetric? NO (1R2 and 2R1)
- Transitive? NO (5R4, 4R2 but 5-2=3 which is not ≤ 2)

Exercise List (1-8)

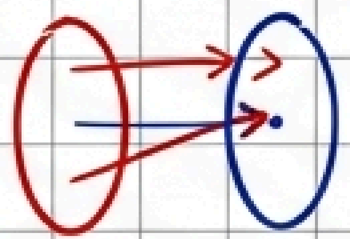
1. Show $A \cap B = A \cap C$ without B=C.
2. Prove $(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$.
3. Partition checks on $\mathbb{N}$: a) $\{n>5\}$, $\{n<5\}$ (No), b) Includes 0 (No), c) n^2 >/< 11 (Yes).
4. Find Sym Difference ($\oplus$) for sets A,B,C,D.
5. 'x divides y' on {1,2,3,4,6}: List pairs, Graph, Matrix, Inverse.
6. Give examples: Sym & Anti-sym; Neither.
7. Prove ad=bc is equivalence on $A \times A$.
8. Prove R^{-1} is equivalence if R is.

4. Functions & 5. Binary Operations

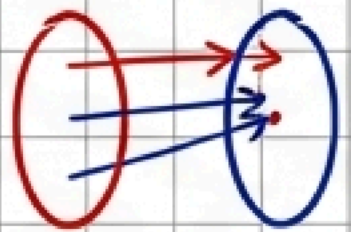
Functions

Definition: Maps every element of domain to **exactly one** element of codomain.

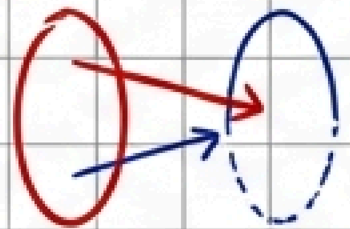
One-to-one (Injective)



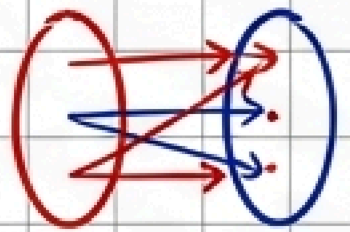
Onto (Surjective)



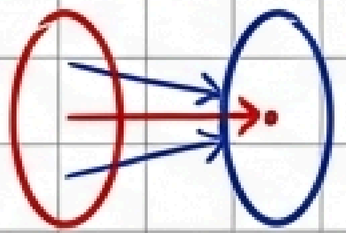
Into



Bijective (1-1 + Onto)



Constant Function



Composite: $(f \circ g)(x) = f(g(x))$

Binary Operations

Operation on Set S: $* : S \times S \to S$.

Properties:

- Commutative
- Associative
- Distributive

Unit II: Trigonometry & Unit III: Cartesian System

Trigonometry Formulas

Angles: $1° = \frac{\pi}{180}$ rad. (π rad = 180°).

Ratios: $\sin = \frac{Opp}{Hyp}$, $\cos = \frac{Adj}{Hyp}$, $\tan = \frac{Opp}{Adj.}$.

tan = Adj.

Reciprocals: cosec, sec, cot.

Identities: $\sin^2\theta + \cos^2\theta = 1$;
$1 + \tan^2\theta = \sec^2\theta$;
$1 + \cot^2\theta = \csc^2\theta$.

Sum of Angles:
$\sin(A+B) = \sin A\cos B + \cos A\sin B$.

Cartesian Coordinates

Distance: $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$.

Section Formula (Internal):

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right).$$

Area of Triangle:

$$\frac{1}{2}\left|x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\right|.$$

Slope: $m = \frac{y_2-y_1}{x_2-x_1}$.

Unit IV: Straight Lines & Unit V: Circles

Straight Lines (Forms of Equation)

Slope-Intercept: $y = mx + c$

Point-Slope: $y - y_1 = m(x - x_1)$

- Two-Point Form
- Intercept Form
- Normal Form

Concurrency: Three lines are concurrent if determinant = 0.

Circles

Standard Equation: $(x-h)^2 + (y-k)^2 = r^2$

General Form: $x^2 + y^2 + 2gx + 2fy + c = 0$

Centre = $(-g, -f)$

Radius = $\sqrt{g^2 + f^2 - c}$

Unit VI: Polynomials & Unit VII: Quadratics

Polynomials

Expression: $a_nx^n + a_{n-1}x^{n-1} + \ldots + a_0$

Types: Monomial, Binomial,

- Trinomial.

Degree: Highest power of x.

Operations: Addition,
Subtraction,
Multiplication.

Quadratic Equations

Standard Form: $ax^2 + bx + c = 0$

Solution Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Nature of Roots (Discriminant D):

- $D > 0 \rightarrow$ Real & distinct
- $D = 0 \rightarrow$ Equal roots
- $D < 0 \rightarrow$ Imaginary

Unit IX: Determinants & Unit X: Combinatorics

Determinants & Matrices

Inverse of Matrix: $A^{-1} = \frac{1}{|A|} Adj(A)$.

Determinant of 2x2: $|A| = ad - bc$.

ad – bc.

Minors & Cofactors: Used to find ad joint and inverse.

Permutation & Combination

Factorial: $n! = n(n-1)(n-2)\ldots1$

Permutation: ${}^nP_r = \frac{n!}{(n-r)!}$

Combination: ${}^nC_r = \frac{n!}{r!(n-r)!}$

Mathematical Notation & Symbols Legend

Set Theory

$\in$: Element of
$\subseteq$: Subset
$\subset$: Proper subset
$\cup$: Union
$\cap$: Intersection
$\emptyset$: Empty Set
A^c : Complement
$\mathbb{P}(A)$: Power Set
$|A|$: Cardinality

Logic & Relations

$\forall$: For all
$\exists$: There exists
$\Rightarrow$: Implies
$\Leftrightarrow$: If and only if
$\times$: Cartesian Product
$\mathbb{R} \circ S$: Composition
$\mathbb{Z}^+$: Positive Integers
$\mathbb{N}$: Natural Numbers

Good Luck!

notes by
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